

BOUNDARY OF TRANSITION FROM QUASISTATIC TO DYNAMIC REGIME  
OF VAPOR BUBBLE GROWTH AND SEPARATION

Yu. A. Kirichenko and N. S. Shcherbakova

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A method is proposed for evaluating the boundary between quasistatic and dynamic regimes of vapor bubble growth and separation. The pressure boundary is evaluated on the basis of data on the boiling of cryogenic liquids and water.

It is known that the character of separation of vapor bubbles from a heat-emitting surface during the nucleate boiling of liquids depends on the corrected pressure [1, 2]. Formulas for the radius of the bubbles at separation in the case of low pressures are determined by solving a dynamic problem, while the formulas for the case of high pressures are found by solving a problem of hydrostatics without allowance for dynamic effects.

The boundary between the dynamic and quasistatic regimes of bubble separation determines the region of applicability of the respective formulas and the boundary between the "low" and "high" pressures for boiling. The presence of such a boundary is seen in analyzing the dependences of internal boiling characteristics on pressure. For example, the dependence of bubble separation radius  $R_d$  on pressure changes its character at  $p/p_b \approx 0.01-0.02$ , while the frequency of separation of bubbles from the heat-emitting surface  $f$  at  $p/p_b \leq 0.005$  increases with an increase in pressure and at  $p/p_b \geq 0.05$  decreases with an increase in pressure [1]. The difference in the functions  $R_d(p)$  and  $f(p)$  at low and high pressures can be attributed to the predominant effect on the vapor bubbles of the forces associated with the inertial reaction of the liquid and surface tension, respectively [3].

We will evaluate the boundary value of pressure  $p_*$  from the corresponding boundary value of the temperature head  $\Delta T_*$ , which is found from comparison of the main forces acting on the vapor bubbles. As the bubbles grow, they are acted upon by buoyancy and by forces which keep the bubbles at the heating surface [3]: surface tension

$$F_\sigma = 2\pi\sigma R_c \quad (1)$$

and the inertial reaction force of the liquid

$$F_I = \frac{\pi}{3} \rho \beta^4. \quad (2)$$

The radius of the microcavity  $R_c$  from which the bubble originates can in a first approximation be assumed to be proportional to the critical radius of the vapor nucleus

$$R_c = B \frac{2\sigma T_s}{L\rho_v \Delta T}, \quad (3)$$

where, according to empirical data, the numerical coefficient is on the order of 10 [2]. Supposing that the bubble grows in accordance with the law  $R = \beta \tau^{1/2}$ , we write the bubble-growth modulus  $\beta$  as follows

$$\beta = C_\beta \left( \frac{\lambda \Delta T}{\rho_v L a} \right)^{n_\beta} a^{1/2}, \quad (4)$$

where for low pressures  $C_\beta = 2\sqrt{\pi/3}$ ,  $n_\beta = 1$  [4] and for high pressures  $C_\beta \approx 4$ ,  $n_\beta = 0.5$  [5].

We will take as the boundary value that value of  $\Delta T_*$  which at the given  $p$  ensures the equality  $F_\sigma = F_I$  (conversely, we will consider the boundary value of  $\Delta p_*$  to be that pressure which corresponds to the thus-determined  $\Delta T_*$ ). In the evaluation, we may take the following for the drag acting on the growing bubble [3]

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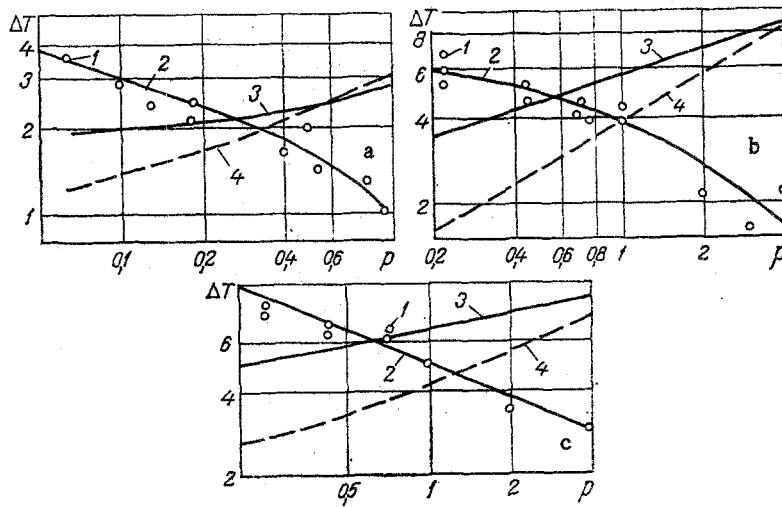


Fig. 1. Dependence of the temperature heads  $\Delta T_0$  and  $\Delta T_*$  on pressure for hydrogen (a), nitrogen (b), and oxygen (c): a)  $\Delta T_0$ : 1) data from [6]; 2) averaged curve from the data in [6];  $\Delta T_*$ : 3) calculation of  $\beta$  in [5]; 4) calculation of  $\beta$  in [4]; b)  $\Delta T_0$ : 1) data from [7]; 2) averaged curve from the data in [7];  $\Delta T_*$ : 3) calculation of  $\beta$  in [5]; 4) calculation of  $\beta$  in [4]; c)  $\Delta T_0$ : 1) data from [7]; 2) averaged curve from the data in [7];  $\Delta T_*$ : 3) calculation of  $\beta$  in [5]; 4) calculation of  $\beta$  in [4].  $\Delta T$ , °K;  $p$ ,  $10^5$  Pa.

TABLE 1. Pressure Boundary between Quasistatic and Dynamic Regimes

Liquid	$\beta$ from [5]		$\beta$ from [4]		$\frac{p_*}{p_b}$
	$p_*$ , $10^5$ Pa	$p_*/p_b$	$p_*$ , $10^5$ Pa	$p_*/p_b$	
Hydrogen [6]	0,28	0,021	0,35	0,027	0,024
Nitrogen [7]	0,55	0,016	1,0	0,029	0,0225
Oxygen [7]	0,67	0,013	1,2	0,024	0,0185
Water [8]	5,3	0,024 0,018	4,0	0,018 0,024	0,021 0,022

$$F_v = 10\pi v \rho \beta^2. \quad (5)$$

However, comparison of Eqs. (2) and (5) shows that  $F \ll F_I$  and that for approximate calculations the quantity  $F_v$  can be ignored, which simplifies our calculations considerably.

Having equated Eqs. (1) and (2) with allowance for (3) and (4), we obtain

$$\Delta T_* = \left( \frac{12B}{C_\beta^2} \right)^{\frac{1}{4n\beta+1}} \left( \frac{\sigma^2 T_s}{\rho} \right)^{\frac{1}{4n\beta+1}} \frac{(L\rho_v)^{\frac{4n\beta-1}{4n\beta+1}} a^{\frac{4n\beta-2}{4n\beta+1}}}{\lambda^{\frac{4n\beta}{4n\beta+1}}}. \quad (6)$$

The intersection of the curve of  $\Delta T_*(p)$  with the curve of the beginning of boiling  $\Delta T_0(p)$  gives the boundary value of  $p_*$  for the beginning of boiling (for the regime of single bubbles). It should be noted that the overwhelming majority of experimental and theoretical studies of vapor bubble growth and separation have been conducted specifically for single, noninteracting bubbles.

Figure 1 shows the relation of  $\Delta T_*$  for several cryogenic liquids calculated from Eq. (6) for  $B = 10$  and from the growth models of Labuntsov [5] and Plesset-Swick [4]. The same figure shows the experimental functions  $\Delta T_0(p)$ . The boundary value of the absolute  $p_*$  and corrected  $p_*/p_b$  is shown in Table 1, which also contains data for water determined in a similar manner. Here, the estimation with calculation of  $\beta$  from the Labuntsov formulas gives an approach from the direction of the quasistatic regime, for which this formula is also valid. Calculation using the Plesset-Swick formula gives an approach from the direction of the dy-

dynamic regime [2]. The set of values of  $p$  and  $\Delta T$  between these two curves can be grouped as a transitional regime. It is evident that the different methods of estimation lead to similar values of boundary pressures: it can be assumed that on the average  $p_* = (0.02 \pm 0.01)p_b$ , as was proposed in [2].

The value of  $\Delta T_*$  can be estimated more accurately if it is determined not from the condition  $F_I = F_\sigma$  but rather from the condition  $F_I = AF_\sigma$ , where  $A \gg 1$  to estimate the boundary of the dynamic regime and  $A \ll 1$  to evaluate the quasistatic regime. For approximate calculations it is evidently sufficient to consider the coefficient  $A$  equal respectively to 3 and 1/3, which produces a 25% shift in the boundary when  $\beta$  is evaluated after Plesset-Swick and 44% when it is evaluated after Labuntsov. This does not significantly affect the result obtained. Whereas the extreme values of  $p_*/p_b$  shown in the table are equal to 0.013 and 0.029, in the more accurate evaluation they would change by 0.006 and 0.035. The latter values, incidentally, are near those which delimit the intermediate region of the pressure dependence on bubble separation frequency or growth time. These quantities change only slightly in this region [1, 2].

Thus, for the regime of single bubbles, the boundary value of pressure at which dynamic or static forces begin to predominate is  $p_*/p_b \approx 0.02$ . At  $p_*/p_b \leq 0.005$ , the regime of bubble growth and separation can be considered purely dynamic. At  $p_*/p_b > 0.03-0.04$ , it can be considered quasistatic. In particular, it follows from this that in analyzing vapor bubbles of water boiling under atmospheric pressure ( $p/p_b \approx 0.0045$ ), one cannot use the formulas obtained by solving static bubble-growth problems — such as the Fritz formula — as is done in the literature.

It should be noted that at  $\Delta T > \Delta T_0$ , the boundary between the dynamic and quasistatic regimes of bubble growth and separation will be shifted in the direction of higher pressures. In the figures all of the point lying to the left and above the curves of  $\Delta T_*(p)$  correspond to conditions whereby there is a dynamic regime (region of low pressures). The points lying to the right and below correspond to a quasistatic regime (region of high pressures). The differences in the dependences not only of the internal, but also the integral characteristics of boiling on the regime parameters at high and low pressures — such as in the relations between heat-transfer coefficient and heat flux and pressure — are also evidently connected with differences in the dynamics of the vapor bubbles on both sides of the boundary [9].

#### NOTATION

$A, B, C_\beta$ , coefficients;  $\alpha$ , diffusivity,  $m^2/sec$ ;  $F_I, F_\sigma, F_v$ , forces, N;  $L$ , heat of vaporization, J/kg;  $n, n_\beta$ , exponents;  $p$ , pressure, Pa;  $T_s, T_q$  saturation temperature and temperature of the heat-emitting surface, °K;  $\Delta T = T_a - T_s$ , temperature head, °K;  $R$ , radius, m;  $f$ , frequency of separation,  $sec^{-1}$ ;  $\beta$ , growth modulus,  $m/sec^{1/2}$ ;  $\lambda$ , thermal conductivity, W/m-deg;  $\sigma$ , surface tension, N/m;  $\rho, \rho_v$ , density of liquid and vapor,  $kg/m^3$ ;  $\nu$ , kinematic viscosity,  $m^2/sec$ ;  $\tau$ , time, sec. Indices: 0, beginning of boiling; \* boundary between quasistatic and dynamic regimes;  $b$ , critical point;  $d$ , separation;  $c$ , cavity.

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HEAT TRANSFER OF A VERTICAL BUNDLE OF HEAT-RELEASING RODS  
IN THE ABSENCE OF CIRCULATION OF THE HEAT CARRIER

M. A. Gotovskii, E. D. Fedorovich,  
V. N. Fromzel', and V. A. Shleifer

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A method is presented for approximate calculation of conductive-radiative heat transfer in bundles of heat-releasing rods, and an empirical estimate is given of the effective thermal conductivity.

It has recently become necessary to develop methods of calculating heat transfer in bundles of heat-releasing rods in the absence of circulation of the heat carrier (coolant). This problem has arisen in connection with the storage and transport of spent fuel assemblies.

Below we examine a system of vertically positioned fuel rods placed in a shell. The greatest difficulty in calculating the temperature regime is presented by allowing for the effect of natural convection. This problem can be solved only by using experimental data. In the limiting case of the absence of natural convection, if we ignore end effects, we come to a two-dimensional problem of conductive-radiative heat transfer. Its solution in an exact formulation presents serious problems in connection with the exceptional awkwardness of the calculations.

To approximately solve the above problem, we will assume that the thermal conductivity of the rods is great enough so that we can assume a constant temperature about the perimeter. We will also assume that the temperature of all of the rods in one row (Fig. 1) is the same, which allows us to examine heat flow only from one row to another. These simplifications made it possible, without serious complications, to superimpose the conductive and radiative heat flows.

First we will examine the radiative heat flow. If we move in the direction of the heat flow, we find that the rods of each row  $i$  interact mainly with the rods of the row  $i + 1$ . The rods of row  $i$  also interact with the rods of row  $i + 2$  although with considerably lower values of reciprocal surface. These three rows constitute an open system of three graybodies. The radiative heat-transfer problem in such a system is fairly complex. We will therefore introduce a simplification which consists mainly of the assumption that the row  $i + 1$  can be approximately regarded as a shield between the  $i$ -th and  $i + 2$ nd rows. Supposing that the shield reduces heat flux by a factor of two, instead of examining the interaction between the  $i$ -th and  $i + 2$ nd rows we will examine the interaction between the  $i$ -th and  $i + 1$ st rows, but we will have increased the reciprocal surface  $H_{1,i+2}$  by a factor of two and added it to the reciprocal surface  $H_{1,i+1}$ . This permits us to reduce the problem to examination of the interaction of two graybodies. Using the familiar relations from [1] to solve this problem, we obtain the following formula for the radiant heat flow between the rows for the condition of equality of the emissivities of the radiating surfaces:

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I. I. Polzunov Scientific-Research Association for the Study and Design of Power-Plant Equipment, Leningrad. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 46, No. 4, pp. 549-554, April, 1984. Original article submitted December 30, 1983.